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Oscillations During Inflation and the Cosmological Density Perturbations

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Abstract

Adiabatic (curvature) perturbations are produced during a period of cosmological inflation that is driven by a single scalar field, the inflaton. On particle physics grounds – though – it is natural to expect that this scalar field is coupled to other scalar degrees of freedom. This gives rise to oscillations between the perturbation of the inflaton field and the perturbations of the other scalar degrees of freedom, similar to the phenomenon of neutrino oscillations. Since the degree of the mixing is governed by the squared mass matrix of the scalar fields, the oscillations can occur even if the energy density of the extra scalar fields is much smaller than the energy density of the inflaton field. The probability of oscillation is resonantly amplified when perturbations cross the horizon and the perturbations in the inflaton field may disappear at horizon crossing giving rise to perturbations in scalar fields other than the inflaton. Adiabatic and isocurvature perturbations are inevitably correlated at the end of inflation and we provide a simple expression for the cross-correlation in terms of the slow-roll parameters.

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1 Introduction

It is commonly believed that the Universe underwent an early era of cosmological inflation. The flatness and the horizon problems of the standard big bang cosmology are elegantly solved if, during the evolution of the early Universe, the energy density is dominated by some vacuum energy and comoving scales grow quasi-exponentially. The prediction of the simplest models of inflation is a flat Universe, *i.e.* $\Omega_{tot} = 1$ with great precision.

Inflation [1] has also become the dominant paradigm for understanding the initial conditions for structure formation and for Cosmic Microwave Background (CMB) anisotropy generation. In the inflationary picture, primordial density and gravity-wave fluctuations are created from quantum fluctuations "redshifted" out of the horizon, where they are "frozen" as perturbations in the background metric [2, 3, 4, 5, 6]. Metric perturbations at the surface of last scattering are observable as temperature anisotropy in the CMB.

Primordial perturbations can be of two kinds, the adiabatic and the isocurvature ones. Recently a lot of attention has been drawn on *correlated* mixtures of the two [7, 8]. In particular in Ref. [8] it has been shown how the correlation between the adiabatic and the isocurvature mode gives rise to new features both in the CMB anisotropies and in the large scale structure. These scenarios are strongly different from those usually consid ered up to now, in which only independent mixtures of the two modes were considered.

Analyses of the possible constraints on these perturbations coming from the present data [9] and future experiments [10] have been also made, as well as investigations on the production of the correlation during inflatio n: in Ref. [7] a specific model of double inflation was considered, and in Ref. [11] a transparent formalism for studying the adiabatic and isocurvature modes was introduced. Here we pursue further the investigation on the possible correlation mechanism, trying to be as general as possible.

Our starting point is the simple observation that, if the inflaton field couples to other scalar degrees of freedom, oscillations between the perturbation of the inflaton field and the perturbations of the other scalar degrees of freedom are induced. For this phenomenon to happen, it is sufficient that the mass squared matrix of the scalar degrees of freedom is not diagonal. This induces a mixing among the different scalar states and such a mixing can be large even if the energy density of the inflaton field dominates over the energy density of the other scalars. We will show that the probability of oscillation is resonantly amplified when perturbations cross the horizon. Adiabatic and isocurvature perturbations are therefore correlated at the end of inflation and we provide a simple expression for the cross-correlation in terms of the slow-roll parameters.

The plan of the paper is as follows. In Section 2 we show that, during an inflationary period in which several scalar fields are present, is natural to expect a mixing and consequent oscillations between the fluctuations of the scalar fields. In Sectio n 3 the correlation of the adiabatic and isocurvature perturbations is explained in terms of this oscillation mechanism, and an explicit expression for it is derived. Finally, in Section 4, we show how to set the initial conditions for structure formation in the post-inflationary epoch in the case where the isocurvature perturbations and the correlation are present.

2 Oscillations during inflation

Despite the simplicity of the inflationary paradigm, the number of inflation models that have been proposed in the literature is enormous [12]. Models have been invented which predict non-Gaussian density fluctuations [13], isocurvature fluctuation modes [14], and cosmic strings [15].

The simplest possibility is represented by the so-called single-field models of inflation, where the the inflationary epoch can be described by a single dynamical order parameter, the inflaton field ϕ. Quantum fluctuations of the inflaton field produce Gaussian adiabatic perturbations of the metric with a nearly scale independent spec-

trum, $n_S \simeq 1$. The amplitude of the perturbation can be characterized by the comoving curvature perturbation $\mathcal{R}$, which remains constant on super-Hubble scales until the perturbation comes back within the Hubble scale long after inflation has ended. Single models of inflation have already been started to be constrained by the recent accurate measurements of the CMB anisotropy [17].

On particle physics grounds – though – it is hard to believe that only one single scalar field ϕ plays a role during the inflationary stage. On the contrary, it is quite natural to expect that during the inflationary dynamics several other scalar fields χ_I ($I = 1, \cdots, N$) are present. As soon as one considers more than one scalar field, one must also consider the role of isocurvature fluctuations [18]. Such perturbations produce an anisotropy in the Cosmic Microwave Background radiation which is six times larger than the adiabatic perturbations. To obtain small CMB anisotropy and still explain galaxy formation one has to strongly suppress isocurvature perturbations on the horizon scale while keeping them sufficiently large on galactic scales. This can be done if the additional scalar fields acquire a mass of the order of the Hubble rate H during inflation. Furthermore, the presence of more than one scalar field may not only affect the evolution of the curvature perturbation, but also give rise to the possibility of seeding isocurvature perturbations after inflation.

The contribution to the total energy density of the extra scalar degrees of freedom χ_I might or might not be negligible compared to the one provided by the scalar field ϕ. If it is, the model of inflation is called multiple field model, a general formalism to evaluate the curvature perturbation at the end of inflation in such models was developed in Ref. [19].

However, even if the contribution to the total energy density of the scalar fields χ_I is small, quantum fluctuations $\delta\chi_I$ of the scalar fields are amplified by gravitational effects. To compute the amplitude and the spectral shape of such perturbations one may consider the theory of a single free scalar field $\delta\chi_I$ with mass m in a de Sitter

background [20].

In this section we wish to show that the generation of the quantum fluctuations in the fields χ_I may be due to another mechanism, which we call *oscillation mechanism* and it is important to stress that such a novel mechanism operates even if the energy density of the scalar field χ_I is much smaller than the contribution coming from a single scalar field ϕ which – for such a reason – deserves the name of inflaton field.

If the scalar sector is composed by more than one single scalar field ϕ, all the scalar degrees of freedom will in general mix. In the particle physics language this can be translated by saying that the mass squared (or the Hamiltonian) in the basis (ϕ, χ_I) is not diagonal or, equivalently, that the states (ϕ, χ_I) are interaction eigenstates, but not mass eigenstates. If the Hamiltonian of two quantum states is not diagonal, the interactions eigenstates (ϕ, χ_I) oscillate during the time evolution of the system. A familiar example in particle physics is represented by the oscillating system of kaons K^0 and $\bar{K}^0$.

The oscillation mechanism responsible for the amplification of the quantum fluctuations of the fields χ_I is due to the oscillations of the inflaton fluctuations into fluctuations of the scalar fields χ_I, in the presence of the inflaton background field ϕ_0. In other words, a perturbation in the inflaton field ϕ may evolve (oscillate) into a perturbation of another scalar degree of freedom χ with a calculable probability.

From a pure quantum mechanical point of view, such fluctuations are generated as coherent states. Therefore the oscillation mechanism is a coherent production of quantum fluctuations[1]. These oscillations during inflation show a behaviour similar to the one present in the phenomenon of coherent neutrino oscillations in a medium like the Sun or the Earth. As we shall see, oscillations during inflation are charecterized by amplification effects analogous to the MSW effect of solar neutrinos.

Before launching ourselves into the details, let us now give a couple of examples

[1] The importance of coherent production of particles after inflation during the preheating stage has been recently emphasized in Ref. [21].

supporting the fact that a large mixing between different scalar fields is generically expected during the inflationary stage. In supergravity and (super)string models there exists a plethora of scalar fields – loosely called moduli – with gravitational-strength couplings to ordinary matter. The mass of these scalar fields is of the order of the gravitino mass $m_{3/2} \sim 100$ GeV in the present vacuum, but is of the order of the Hubble rate H during inflation. In these theories coupling constants and masses appearing in the Lagrangian have to be thought as functions of the dimensionless ratio $\chi_I/M_{\rm Pl}$, where χ_I $(I = 1, \cdots, N)$ denotes a generic modulus field and $M_{\rm Pl}$ is the Planck mass. For instance, a generic coupling constant λ is in fact a function of the scalar moduli

$$\lambda(\chi_I) = \lambda \left(\frac{\langle \chi_I \rangle}{M_{\rm Pl}}\right) \left(1 + c\frac{\delta\chi_I}{M_{\rm Pl}} + \cdots\right), \tag{1}$$

where c is a coefficient usually of order unity and $\delta\chi_I = \chi_I - \langle \chi_I \rangle$. This expansion introduces a direct coupling between the scalar moduli and the inflaton. The potential V becomes a function of two (or more) fields, $V = V(\phi, \chi_I)$ and the second derivative $(\partial^2 V/\partial\phi\partial\chi_I)$ may be as large as H^2 during inflation. If we set $\phi \equiv \chi_0$, all the elements of the mass squared matrix $\mathcal{M}^2_{ij} \equiv (\partial^2 V/\partial\chi_i\partial\phi_j)$ $(j = 0, \cdots, N)$ are of the form $c_{ij}\, H^2$ where $c_{ij} = \mathcal{O}(1)$. A considerable mixing between the inflaton field and the moduli fields is generated if the inflaton background field ϕ_0 takes values as large as the Planck scale. Notice that this may happen even if $c_{00} \ll 1$, as required by the flatness of the inflaton potential. Under these circumstances, the perturbation of the scalar field ϕ may oscillate into a perturbation of the modulus field which is generated as a coherent state.

Another example may be provided by theories in which gravity may propagate in extra-dimensions [22] where there appears a infinite tower of spin-0 graviscalar Kaluza-Klein excitations and the inflaton field can mix to these particles by coupling to the higher-dimensional Ricci scalar.

Let us now describe a simple, but illustrative example. Consider two scalar fields, ϕ and χ. We will dub ϕ the inflaton field, even if this might be a misnomer as the two

fields might give a comparable contribution to the total energy density of the Universe.

The scalar field perturbations, with comoving wavenumber $k = 2\pi a/\lambda$ for a mode with physical wavelength λ, obey the perturbation equations

$$\begin{aligned} \delta\ddot{\phi} + 3H\delta\dot{\phi} + \frac{k^2}{a^2}\delta\phi + V_{\phi\phi}\delta\phi + V_{\phi\chi}\delta\chi &= 0 \\ \delta\ddot{\chi} + 3H\delta\dot{\chi} + \frac{k^2}{a^2}\delta\chi + V_{\chi\chi}\delta\chi + V_{\chi\phi}\delta\phi &= 0, \end{aligned} \qquad (2)$$

where we have indicated by $V_{\phi\phi} = (\partial^2 V/\partial\phi\partial\phi)$ and similar notation for the other derivatives.

The squared mass matrix is given by

$$\mathcal{M}^2 = \begin{pmatrix} V_{\phi\phi} & V_{\phi\chi} \\ V_{\phi\chi} & V_{\chi\chi} \end{pmatrix}. \qquad (3)$$

We now introduce a time-dependent 2×2 unitary matrix $\mathcal{U}$ such that

$$\mathcal{U}^\dagger \mathcal{M}^2 \mathcal{U} = \mathrm{diag}\,(\omega_1^2, \omega_2^2) \equiv \omega^2. \qquad (4)$$

In the following we will assume that all the entries of the squared mass matrix $\mathcal{M}^2$ are real, so that the unitary matrix $\mathcal{U}$ reduces to an orthogonal matrix

$$\mathcal{U} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}, \qquad (5)$$

where

$$\tan 2\theta = \frac{2\,V_{\chi\phi}}{V_{\phi\phi} - V_{\chi\chi}} \qquad (6)$$

and the mass eigenvalues are given by

$$\omega_{1,2}^2 = \frac{1}{2}\left[(V_{\phi\phi} + V_{\chi\chi}) \pm \sqrt{(V_{\phi\phi} - V_{\chi\chi})^2 + 4\,V_{\chi\phi}^2}\right]. \qquad (7)$$

Adopting the vectorial notation $\Psi = (\Psi_1, \Psi_2)^T = \mathcal{U}^T(\phi, \chi)^T$, the equations of the scalar perturbations may be rewritten as

$$\delta\ddot{\Psi} + \left(3H + 2\mathcal{U}^T\dot{\mathcal{U}}\right)\delta\dot{\Psi} + \left(\frac{k^2}{a^2} + \omega^2 + 3\,H\,\mathcal{U}^T\dot{\mathcal{U}} + \mathcal{U}^T\ddot{\mathcal{U}}\right)\delta\Psi = 0, \qquad (8)$$

where

$$\mathcal{U}^T\dot{\mathcal{U}} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \dot{\theta} \tag{9}$$

and

$$\mathcal{U}^T\ddot{\mathcal{U}} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \ddot{\theta} + \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \dot{\theta}^2. \tag{10}$$

Notice that the matrix $\mathcal{U}$ diagonalizes the squared mass matrix at the price of introducing further non-diagonal terms in the equation of motion.

To proceed further, we may now take advantage of the slow-roll conditions which are to be attained during inflation [12]. If we consider the generic slow-roll parameters

$$\varepsilon_{ij} = \frac{1}{2}\frac{M^2 V_i V_j}{V^2} \quad \text{and} \quad \eta_{ij} = M^2 \frac{V_{ij}}{V}, \tag{11}$$

where $M = M_{\rm Pl}/\sqrt{8\pi}$ is the reduced Planck mass, a successfull period of inflation requires that $|\varepsilon_{ij}, \eta_{ij}| \ll 1$, *i.e.* the potential has to be flat enough for inflation to develop.

Since time derivatives of the slow-roll parameters are second-order in the slow-roll parameters themselves, $\dot{\varepsilon}, \dot{\eta} \sim \mathcal{O}(\varepsilon^2, \eta^2)$, it is easy to convince oneself that – if we keep only the slow-roll paramaters at the first-order – Eq. (8) gets simplified to

$$\delta\ddot{\Psi} + 3H\delta\dot{\Psi} + \left(\frac{k^2}{a^2} + \omega^2 + 3\,H\,\mathcal{U}^T\dot{\mathcal{U}}\right)\delta\Psi = 0. \tag{12}$$

Introducing the conformal time $d\tau = dt/a$, where a is the scale factor of the expanding Universe, and the rescaled fields $\delta\widetilde{\Psi}_1 = a\delta\Psi_1$ and $\delta\widetilde{\Psi}_2 = a\delta\Psi_2$, Eq. (12) becomes

$$\begin{aligned} \delta\widetilde{\Psi}_1'' + \left(k^2 - \frac{a''}{a} + \omega_1^2 a^2\right)\delta\widetilde{\Psi}_1 + 3H\,\theta' a\,\delta\widetilde{\Psi}_2 &= 0 \\ \delta\widetilde{\Psi}_2'' + \left(k^2 - \frac{a''}{a} + \omega_2^2 a^2\right)\delta\widetilde{\Psi}_2 - 3H\,\theta' a\,\delta\widetilde{\Psi}_1 &= 0 \end{aligned} \tag{13}$$

To illustrate the phenomenon of oscillations during inflation, we make the assumption that the non-diagonal terms in Eq. (13) proportional to $3H\theta' a = 3H\dot{\theta}a^2$ are smaller than the diagonal entries $\omega_{1,2}^2 a^2$. This hypothesis is correct, for instance, if the squared masses are constant in time. We adopt this simplification in order to render the description of the phenomenon of oscillations during inflation more transparent.

In fact, in the following section we will solve the problem exactly at the first-order in the slow-roll parameters and show that at this order of approximation taking $3H\theta' a \ll \omega_{1,2}^2$ does not change the final result for the probability.

Eq. (13) is solved by[2]

$$\delta\widetilde{\Psi}_i = \frac{1}{2}\sqrt{\pi}\, e^{i\left(\nu_i+\frac{1}{2}\right)\frac{\pi}{2}}\,(-\tau)^{1/2}\, H^{(1)}_{\nu_i}(-k\tau), \;\; i=1,2, \tag{14}$$

where the conformal time τ assumes negative values (the beginning of inflation is at some $|\tau| \gg 1$), $H_\nu^{(1)}$ are the Hankel's functions of the first kind and $\nu_i^2 = 9/4 - (\omega_i/H)^2$. The normalization factor in Eq. (14) is chosen such that $\delta\widetilde{\Psi}_i$ matches the plane-wave $e^{-ik\tau}/\sqrt{2k}$ for subhorizon scales in the far ultraviolet $k/aH \gg 1$.

We are now in the position to ask what is the probability (as a function of time) that a scalar perturbation in the "inflaton" field $\delta\phi$ becomes a scalar field perturbation in the scalar field $\delta\chi$. The answer to this question is readily given if we remember that $[\phi(\tau), \chi(\tau)]^T = \mathcal{U}[\Psi_1(\tau), \Psi_2(\tau)]^T$. This means that – at a given time τ – the scalar perturbations $\delta\phi$ and $\delta\chi$ are a linear combination of the mass eigenstates scalar perturbations $\delta\Psi_1$ and $\delta\Psi_2$

$$\delta\phi = \sum_{\ell=1,2} \mathcal{U}_{1\ell}\,\delta\Psi_\ell, \;\; \delta\chi = \sum_{\ell=1,2} \mathcal{U}_{2\ell}\,\delta\Psi_\ell. \tag{15}$$

The probability that a scalar perturbation $\delta\phi$ at the time τ_0 becomes a scalar perturbation $\delta\chi$ at the time τ is therefore given in general by

$$P\left[\delta\phi(\tau_0) \to \delta\chi(\tau)\right] = \left| \sum_{\ell=1,2} \mathcal{U}^*_{1\ell}(\tau_0)\,\mathcal{U}_{2\ell}(\tau)\, \frac{\delta\Psi^*_\ell(\tau_0)}{|\delta\Psi^*_\ell(\tau_0)|}\frac{\delta\Psi_\ell(\tau)}{|\delta\Psi_\ell(\tau)|}\right|^2. \tag{16}$$

We take as initial condition $\tau_0 \to -\infty$, and we follow the conversion probability at time τ.

If the unitary matrix $\mathcal{U}$ is real, Eq. (16) becomes

$$P\left[\delta\phi(\tau_0) \to \delta\chi(\tau)\right] = \frac{1}{2}\sin^2 2\theta \left\{ 1 - \frac{\mathrm{Re}\left[\delta\Psi_1^*(\tau_0)\delta\Psi_1(\tau)\delta\Psi_2(\tau_0)\delta\Psi_2^*(\tau)\right]}{|\delta\Psi_1^*(\tau_0)\delta\Psi_1(\tau)\delta\Psi_2(\tau_0)\delta\Psi_2^*(\tau)|}\right\}. \tag{17}$$

[2]To be consistent one should also expand the factor $\frac{a''}{a}$ up to the first order in the slow-roll parameters, reflecting the fact that inflation does not generically occurs with a pure de Sitter dynamics. However, these corrections do not change the final expression for the oscillation probability, see Eq. (20), since they would alter the wave-functions of the two mass eigenstates in an equal manner.

Let us investigate how such probability changes as a function of the physical wavelength $\lambda = 2\pi(a/k)$. At subhorizon scales, $k \gtrsim aH$, the functions $\delta\Psi_k$ tend to the common plane-wave solution $e^{-ik\tau}/\sqrt{2k}$ and therefore

$$P\left[\delta\phi(\tau_0) \rightarrow \delta\chi(\tau)\right] \simeq 0 \ \ (k \gg aH). \tag{18}$$

However, at superhorizon scales, $k \lesssim aH$, the functions $\delta\widetilde{\Psi}_\ell$ develops a phase dependence

$$\delta\widetilde{\Psi}_\ell \simeq e^{i(\nu_\ell-\frac{1}{2})\frac{\pi}{2}}\, 2^{\nu_\ell-\frac{3}{2}}\, \frac{\Gamma(\nu_\ell)}{\Gamma(3/2)}\, \frac{1}{\sqrt{2k}}\, (-k\tau)^{\frac{1}{2}-\nu_\ell} \tag{19}$$

and the conversion probability becomes

$$\begin{aligned} P\left[\delta\phi(\tau_0) \rightarrow \delta\chi(\tau)\right] &= \frac{1}{2}\sin^2 2\theta \left[1 - \cos\left(\frac{\pi}{2}\Delta\nu\right)\right] \\ &= \sin^2 2\theta\, \sin^2\left(\frac{\pi}{4}\Delta\nu\right) \ \ (k \ll aH), \end{aligned} \tag{20}$$

where $\Delta\nu = \nu_1 - \nu_2$.

In the limit $\omega^2_{1,2} \ll H^2$, such a probability reduces to

$$P\left[\delta\phi(\tau_0) \rightarrow \delta\chi(\tau)\right] \simeq \sin^2 2\theta\, \sin^2\left(\frac{\pi}{12}\frac{\Delta\omega^2}{H^2}\right), \tag{21}$$

with $\Delta\omega^2 = \omega_1^2 - \omega_2^2$.

Note that the same formula holds for the probability $P\left[\delta\chi(\tau_0) \rightarrow \delta\phi(\tau)\right]$. This is due to the fact that eq. (16) depends only on the mass eigenstates and it is symmetric in $\delta\widetilde{\Psi}_1$ and $\delta\widetilde{\Psi}_2$.

The expression (20) reminds the well-known formula which describes the evolution in time of the probability of oscillations between two neutrino flavours [23]. In both cases the probability is identically zero if the two mass eigenstates have equal masses (no oscillations are present in the degenerate case); there is the same dependence (as $\sin^2 2\theta$) on the mixing angle θ and the same functional dependence on the difference of the squared masses. Differences are present, though. While the probability of neutrino conversion is depending upon time for any value of the neutrino energy, in our case at

superhorizon scales the conversion probability becomes constant in time. This does not come as a surprise since on superhorizon scales the dynamics of the system is frozen.

What is more interesting is the time evolution of the conversion probability. Consider a scalar perturbation in the inflaton field with a given physical wavelength $\lambda = 2\pi(a/k)$ which gets stretched during inflation. As long as the wavelength remains subhorizon, the scalar perturbation remains a pure perturbation in the inflaton field. However, as soon as the wavelength crosses the horizon, the perturbation in the inflaton field may become (generate) a perturbation in the other scalar field χ with a nonvanishing probability determined by Eq. (20). At horizon crossing there is an amplification mechanism of the fluctuations in the field χ which is reminiscent of the MSW effect operative for solar neutrinos.

The phenomenon of resonant amplification is easily understood if one remembers that a given wavelength crosses the horizon when $k = aH$, *i.e.* when $k^2 = a''/a$ using the conformal time. As long as the wavelength is subhorizon, $k^2 \gg a''/a$, the presence of the mass terms in the equations of motion (13) is completely negligible compared to the factor $(k^2 - a''/a)$. On the other hand, when the wavelength crosses the horizon the term $(k^2 - a''/a)$ vanishes and the effect of the mixing in the mass squared matrix is magnified, giving rise to the resonant effect. Finally, when the wavelength is larger than the horizon, $k^2 \ll a''/a$, the term $(k^2 - a''/a)$ starts to dominate again over the mass terms and the oscillations get frozen.

We conclude that fluctuations in the scalar field χ are generated as coherent states at horizon crossing through the oscillation mechanism out of perturbations in the scalar field ϕ.

A couple of comments are in order here. First of all, we wish to stress that the oscillation mechanism operates even if the energy of the inflaton field ϕ is much larger than the energy stored in the other scalar field χ. This is because what is crucial for the oscillations to occur is the *relative* magnitude of the elements of the mass squared

matrice $\mathcal{M}^2$. Secondly, the magnitude of the probability depends upon two quantities, $\sin^2 2\theta$ and $\Delta\omega^2/H^2$. Both can be readily expressed in terms of the slow-roll parameters. The first factor is not necessarily small, in fact it may be even of order unity for maximal mixing. If expanded in terms of the slow-roll parameters, it is $\mathcal{O}(\eta^0, \epsilon^0)$. The second term is naturally smaller than unity and is linear in the slow-roll parameters. This reflects the fact that during inflation only perturbations in those scalar fields with masses smaller than the Hubble rate may be excited. However, $\Delta\omega^2/H^2$ is not necessarily much smaller than unity and the amplification of the conversion probability at horizon crossing may be sizeable.

3 Correlation between adiabatic and entropy perturbations during inflation

As we already mentioned in the previous section, adiabatic (curvature) and entropy (isocurvature) perturbations are produced during a period of cosmological inflation if more than one scalar field is present at this epoch. In this section we wish to provide a simple expression for the cross-correlation between adiabatic and entropy perturbations inspired by the considerations developed in the previous section.

A consistent study of the linear field fluctuations requires the knowledge of the linear scalar perturbations of the metric, corresponding to the line element

$$\begin{aligned} ds^2 &= -(1+2A)dt^2 + 2aB_{,i}dx^i dt \\ &+ a^2\left[(1-2\psi)\delta_{ij} + 2E_{,ij}\right]dx^i dx^j. \end{aligned} \tag{22}$$

The consistent equation for a generic scalar field perturbation $\delta\chi_I$ $(I = 1, \cdots, N)$ with comoving wavenumber $k = 2\pi a/\lambda$ for a mode with physical wavelength λ reads

$$\begin{aligned} &\ddot{\delta\chi}_I + 3H\dot{\delta\chi}_I + \frac{k^2}{a^2}\delta\chi_I + \sum_J V_{\chi_I\chi_J}\delta\chi_J \\ &= -2V_{\chi_I}A + \dot{\chi}_I\left[\dot{A} + 3\dot{\psi} + \frac{k^2}{a^2}(a^2\dot{E} - aB)\right]. \end{aligned} \tag{23}$$

At this stage it is useful to introduce the gauge-invariant Sasaki-Mukhanov variables [24]

$$Q_I \equiv \delta\varphi_I + \frac{\dot{\varphi}_I}{H}\psi \tag{24}$$

which, in the spatially flat gauge $\psi_Q = 0$, obey the equations of motion

$$\ddot{Q}_I + 3H\dot{Q}_I + \frac{k^2}{a^2}Q_I + \mathcal{M}^2_{IJ}Q_J = 0, \tag{25}$$

where

$$\mathcal{M}^2_{IJ} = V_{\chi_I\chi_J} - \frac{1}{M^2 a^3}\left(\frac{a^3}{H}\dot{\chi}_I\dot{\chi}_J\right)^{\cdot} \simeq \frac{V}{M^2}\left(\eta_{IJ} - \frac{2}{3}\epsilon_{IJ}\right), \tag{26}$$

where the last expression has been obtained performing an expansion up to the first order in the slow-roll parameters. This equation is similar to Eq. (2) and from what we have learned in the previous section, oscillations among the different quantities Q_I are expected to take place. Notice also that the Q_I's oscillate even if the part of the mass squared matrix $\mathcal{M}^2_{IJ}$ proportional to $V_{\chi_I\chi_J}$ is diagonal. This is because non-diagonal entries are always present because of the ϵ_{IJ}-parameters.

Once the variables Q_I have been defined, one can define the comoving curvature perturbation [25, 26]

$$\mathcal{R} = \sum_I \left(\frac{\dot{\varphi}_I}{\sum_{J=1}^N \dot{\varphi}_J^2}\right) Q_I \tag{27}$$

and give a dimensionless definition of the total entropy perturbation (automatically gauge-invariant)

$$S = H\left(\frac{\delta p}{\dot{p}} - \frac{\delta\rho}{\dot{\rho}}\right). \tag{28}$$

For N scalar fields the latter is given by

$$S = \frac{2\left(\dot{V} + 3H\sum_{J=1}^N \dot{\varphi}_J^2\right)\delta V + 2\dot{V}\sum_I \dot{\varphi}_I(\dot{\delta\varphi_I} - \dot{\varphi}_I A)}{3\left(2\dot{V} + 3H\sum_J \dot{\varphi}_J^2\right)\sum_{I=1}^N \dot{\varphi}_I^2}. \tag{29}$$

Let us now restrict ourselves to the case of two fields, ϕ and χ. Following the nice treatement of Ref. [11], we can define two new adiabatic and entropy fields by a rotation in field space. We define the "entropy field" s [11]

$$\delta s = (\cos\beta)\delta\chi - (\sin\beta)\delta\phi, \tag{30}$$

where

$$\cos\beta = \frac{\dot\phi}{\sqrt{\dot\phi^2+\dot\chi^2}}, \quad \sin\beta = \frac{\dot\chi}{\sqrt{\dot\phi^2+\dot\chi^2}}. \tag{31}$$

Notice that δs can be rewritten as

$$\delta s = (\cos\beta) Q_\chi - (\sin\beta) Q_\phi, \tag{32}$$

From this definition it follows that s =constant along the classical trajectory, and hence entropy perturbations are automatically gauge-invariant [27, 11].

The adiabatic part of the perturbation is associated to the orthogonal combination

$$\delta Q_A = (\sin\beta) Q_\chi + (\cos\beta) Q_\phi. \tag{33}$$

Our goal is to give an expression of the cross-correlation between the adiabatic and the entropy perturbations

$$\langle Q_A(k)\delta s^*(k')\rangle \equiv \frac{2\pi^2}{k^3}\,\mathcal{C}_{Q_A\delta s}\,\delta(k-k'). \tag{34}$$

To do so, we adopt the technique developed in the previous section. As we have seen, though, introducing a unitary matrix $\mathcal{U}$ which diagonalizes the mass squared matrix is not enough to diagonalize the full system, (see Eq. (13)). This happens because, in general, the fields are coupled together. To proceed, we first define the comoving fields $\widetilde{Q}_\phi = aQ_\phi$ and $\widetilde{Q}_\chi = aQ_\chi$, then we introduce a basis for annihilation and creation operators a_i and $a_i^\dagger$ $(i=1,2)$ and perform the decomposition (τ is the conformal time)

$$\begin{aligned}
\begin{pmatrix} \widetilde{Q}_\phi \\ \widetilde{Q}_\chi \end{pmatrix} &= \mathcal{U}\int \frac{d^3k}{(2\pi)^{3/2}}\left[e^{i\mathbf{k}\cdot\mathbf{x}}\,h(\tau)\begin{pmatrix} a_1(k) \\ a_2(k)\end{pmatrix} + \text{h.c.}\right], \\
\begin{pmatrix} \Pi_{\widetilde{Q}_\phi} \\ \Pi_{\widetilde{Q}_\chi} \end{pmatrix} &= \mathcal{U}\int \frac{d^3k}{(2\pi)^{3/2}}\left[e^{i\mathbf{k}\cdot\mathbf{x}}\,\widetilde{h}(\tau)\begin{pmatrix} a_1(k) \\ a_2(k)\end{pmatrix} + \text{h.c.}\right],
\end{aligned} \tag{35}$$

where $\Pi_{\widetilde{Q}_\phi}$ and $\Pi_{\widetilde{Q}_\chi}$ are the conjugate momenta of $\widetilde{Q}_\phi$ and $\widetilde{Q}_\chi$ respectively, and h and $\widetilde{h}$ are two 2×2 matrices satisfying the relation

$$\left[h\,\widetilde{h}^* - h^*\,\widetilde{h}^T\right]_{ij} = i\,\delta_{ij}, \tag{36}$$

derived from the canonical quantization condition. The matrix $\mathcal{U}$ is given in Eq. (5). As it diagonalizes the squared mass matrix (26) (with the identification $\chi_1 = \phi$ and $\chi_2 = \chi$), the mixing angle is given by

$$\tan 2\theta = \frac{2\,\mathcal{M}^2_{\chi\phi}}{\mathcal{M}^2_{\phi\phi} - \mathcal{M}^2_{\chi\chi}} \tag{37}$$

and the mass eigenvalues are given by

$$\omega^2_{1,2} = \frac{1}{2}\left[\left(\mathcal{M}^2_{\phi\phi} + \mathcal{M}^2_{\chi\chi}\right) \pm \sqrt{\left(\mathcal{M}^2_{\phi\phi} - \mathcal{M}^2_{\chi\chi}\right)^2 + 4\,\mathcal{M}^2_{\chi\phi}}\right]. \tag{38}$$

It is not difficult to see that the matrix h satisfies the following differential equation

$$h'' + 2\mathcal{U}\mathcal{U}'\,h' + \left[k^2 - \frac{a''}{a} + \omega^2 a^2 + (\mathcal{U}\mathcal{U}')^2 + (\mathcal{U}\mathcal{U}')'\right]\,h = 0, \tag{39}$$

If we now expand up to the first order of perturbation in the slow-roll parameters, we obtain

$$\begin{aligned} h''_{11} + 2\,\theta'\,h'_{21} + \left(k^2 - \frac{a''}{a} + \omega_1^2 a^2\right) h_{11} + H\,\theta' a\,h_{21} &= 0, \\ h''_{21} - 2\,\theta'\,h'_{11} + \left(k^2 - \frac{a''}{a} + \omega_2^2 a^2\right) h_{21} - H\,\theta' a\,h_{11} &= 0, \end{aligned} \tag{40}$$

and similar equations for h_{12} and h_{22} with the substitution $h_{11} \to h_{12}$ and $h_{21} \to h_{22}$.

Making use of the decomposition (35) and definitions (32) and (33), we find that

$$\begin{aligned} a^2\langle Q_A(k)\delta s^*(k')\rangle &= (s_\beta c_\theta - c_\beta s_\theta)\,(c_\beta c_\theta + s_\beta s_\theta)\left[|h_{22}|^2 - |h_{11}|^2 + |h_{21}|^2 - |h_{12}|^2\right] \\ &+ (c_\beta c_\theta + s_\beta s_\theta)^2\,[h_{11}h^*_{21} + h_{12}h^*_{22}] \\ &- (s_\beta c_\theta - c_\beta s_\theta)^2\,[h_{21}h^*_{11} + h_{22}h^*_{12}]\,, \end{aligned} \tag{41}$$

where we have made use of the shorthand notation $c_{\beta(\theta)} = \cos\beta(\theta)$ and $s_{\beta(\theta)} = \sin\beta(\theta)$. To check this expression, we can consider the following limiting cases:

i) No squared mass matrix: If the squared mass matrix (26) is identically zero, Q_ϕ and Q_χ are already orthogonal states and mass eigenstates and their time evolution is identical. This means that there is no correlation between them and one expects

$\langle Q_A(k)\delta s^*(k')\rangle \propto s_\beta c_\beta \left(\langle Q_\phi(k) Q^*_\phi(k')\rangle - \langle Q_\chi(k) Q^*_\chi(k')\rangle\right) = 0$. This is confirmed by Eq. (41) since in this case $c_\theta = 1$, $s_\theta = 0$, $h_{11} = h_{22}$ and $h_{12} = h_{21} = 0$.

ii) Two noninteracting fields: Suppose the system is composed by two fields ϕ and χ with potential $V = \frac{1}{2}m^2(\phi^2 + \chi^2)$. In such a case, $c_\beta = s_\beta = 1/\sqrt{2}$ and the squared mass matrix (26) as the following structure

$$\mathcal{M}^2 \propto \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}, \tag{42}$$

coming from the terms proportional to the ϵ-parameters. This means that the mixing angle $\theta = \pi/4$ and $c_\theta = s_\theta = 1/\sqrt{2}$. The system is already diagonalized by the unitary matrix $\mathcal{U}$ and there is no need for the nondiagonal elements of the matrix h, $h_{12} = h_{21} = 0$. In such a case, the expression (41) predicts that $\langle Q_A(k)\delta s^*(k')\rangle$ vanishes. This confirms the findings of Ref. [7].

To compute the cross-correlation between the adiabatic and the isocurvature modes in a more general way, we can expand the solutions of the system (40) in powers of the slow-roll parameters.

In the far ultraviolet $k \gg aH$ the squared mass matrix is negligible and Q_ϕ and Q_χ are mass eigenstates. Therefore, we set the physical initial conditions by posing the unitary matrix $\mathcal{U}$ proportional to the unity matrix and $h_{ij} = \delta_{ij}\, e^{-ik\tau}/\sqrt{2k}$. A simple inspection of the system (40) tells us that the functions h_{12} and h_{21} are sourced by the functions h_{22} and $-h_{11}$, respectively. Since θ' is already $\mathcal{O}(\eta, \epsilon)$ during the time evolution of the system $h_{12} = -h_{21} = \mathcal{O}(\eta, \epsilon)$. Expanding Eq. (41) up to first order in the slow-roll parameters, we get

$$\begin{aligned} a^2\langle Q_A(k)\delta s^*(k')\rangle &= (s_\beta c_\theta - c_\beta s_\theta)(c_\beta c_\theta + s_\beta s_\theta)\left[|h_{22}|^2 - |h_{11}|^2\right] \\ &+ \left[h_{11} h^*_{21} - h_{21} h^*_{11}\right], \end{aligned} \tag{43}$$

where we have made use of the fact that $|h_{22}|^2 - |h_{11}|^2 = \mathcal{O}(\eta, \epsilon)$.

A simple perturbative procedure shows that

$$h_{21}(\tau) \simeq \int^{\tau} d\xi \, \frac{f(\xi) f^*(\tau) - f^*(\xi) f(\tau)}{W(\xi)} \, \theta'(\xi) \, \left[2\, f'(\xi) + H\, a(\xi)\, f(\xi) \right], \tag{44}$$

where

$$\begin{aligned} f(\tau) &= \frac{1}{2}\sqrt{\pi}\, e^{i\pi}\, (-\tau)^{1/2}\, H^{(1)}_{3/2}(-k\tau), \\ W(\tau) &= f\, f^{*\prime} - f^*\, f'. \end{aligned} \tag{45}$$

As long as the wavelength of the perturbations is subhorizon, $h_{11} = h_{22} \simeq e^{-ik\tau}/\sqrt{2k}$ and solving Eq. (44) gives $|h_{21}| \propto (H/k)\, |h_{11}| \ll |h_{11}|$.

At superhorizon scales $k \ll aH$, h_{11}, h_{22} are proportional to the scale factor (up to terms linear in the slow-roll parameters). Since $\theta' = a\,\dot{\theta} = a\, H\, F(\epsilon, \eta)$, where F is a function linear in the slow-roll parameters, the integral in Eq. (44) can be performed and gives

$$h_{21} = -3\, h_{11}\, F(\epsilon, \eta) \ln \left(\frac{k}{aH} \right). \tag{46}$$

With this information at hand, let us first evaluate the probability that a the perturbation Q_ϕ at a time τ_0 becomes a perturbation Q_χ at a generic time τ. Using the decomposition (35) and making use of the fact that $h_{12} = -h_{21} = \mathcal{O}(\eta, \epsilon)$ during the time evolution of the system, the probability may be written as

$$\begin{aligned} & P\left[Q_\phi(\tau_0) \to Q_\chi(\tau)\right] = \left|\langle Q_\chi(\tau) | Q_\phi(\tau_0)\rangle\right|^2 \\ &= \left| c_\theta\, s_\theta \left[\frac{h_{11}(\tau_0)\, h^*_{11}(\tau)}{|h_{11}(\tau_0)\, h^*_{11}(\tau)|} - \frac{h_{22}(\tau_0)\, h^*_{22}(\tau)}{|h_{22}(\tau_0)\, h^*_{22}(\tau)|} \right] \right|^2 . \end{aligned} \tag{47}$$

Such a probability is vanishing at subhorizon scales, but at superhorizon scale

$$\begin{aligned} P\left[Q_\phi(\tau_0) \to Q_\chi(\tau)\right] &= \frac{1}{2} \sin^2 2\theta \left[1 - \cos\left(\frac{\pi}{2} \Delta\nu \right) \right] \\ &= \sin^2 2\theta\, \sin^2 \left(\frac{\pi}{4} \Delta\nu \right), \end{aligned} \tag{48}$$

which reproduces Eq. (20).

Therefore, we expect that the cross-correlation is extremely small at subhorizon scales reflecting the fact that Q_ϕ and Q_χ are good mass eigenstates Nevertheless, the

cross-correlation does not vanish at superhorizon scales and is given by

$$\mathcal{C}_{Q_A\delta s} = \left(s_\beta c_\theta - c_\beta s_\theta\right)\left(c_\beta c_\theta + s_\beta s_\theta\right)\left(\frac{H_k}{2\pi}\right)^2 \left[1 - \left(\frac{k}{aH}\right)^{-2\Delta\nu}\right], \tag{49}$$

where $H_k = k/a$ and $\Delta\nu = \nu_2 - \nu_1$ (which is given by $-1/3\,\Delta\omega^2/H^2 = -1/3(\omega_2^2 - \omega_1^2)/H^2$ if $\omega_{1,2}^2 \ll H^2$). If we normalize the scale factor a in such a way that $a = 1$ at the end of inflation and indicate by $N = -\ln a$ the number of e-folds until the end of inflation, we finally get

$$\mathcal{C}_{Q_A\delta s} = \left(s_\beta c_\theta - c_\beta s_\theta\right)\left(c_\beta c_\theta + s_\beta s_\theta\right)\left(\frac{H_k}{2\pi}\right)^2 \left[1 - \left(\frac{k}{H}\right)^{\frac{2}{3}\frac{\Delta\omega^2}{H^2}} e^{\frac{2}{3}N\frac{\Delta\omega^2}{H^2}}\right]. \tag{50}$$

This result is remarkably simple and can be easily expressed in terms of the slow-roll parameters using the expressions

$$\frac{\Delta\omega^2}{H^2} \simeq 3\sqrt{\left[\eta_{\phi\phi} - \eta_{\chi\chi} - \frac{2}{3}\left(\epsilon_{\phi\phi} - \epsilon_{\chi\chi}\right)\right]^2 + 4\left(\eta_{\phi\chi} - \frac{2}{3}\epsilon_{\phi\chi}\right)^2}, \tag{51}$$

and

$$\tan 2\theta = \frac{2\left(\eta_{\phi\chi} - \frac{2}{3}\epsilon_{\phi\chi}\right)}{\eta_{\phi\phi} - \eta_{\chi\chi} - \frac{2}{3}\left(\epsilon_{\phi\phi} - \epsilon_{\chi\chi}\right)}. \tag{52}$$

The origin of the cross-correlation is due to a rather transparent physical behaviour. At the inflationary epoch, the gauge invariant perturbations Q_ϕ and Q_χ are generated with different wavelengths stretched by the superluminal expansion of the scale factor. Since the squared mass matrix of Q_ϕ and Q_χ is not diagonal, oscillations between the two quantities are expected. Till the wavelength remains subhorizon, Q_ϕ and Q_χ evolve independently and may be considered good mass eigenstates. However, as soon as the wavelength crosses the horizon, an amplification in the probability of oscillation between Q_ϕ and Q_χ occurs: a nonvanishing correlation between Q_ϕ and Q_χ is created on superhorizon scales because of the nondiagonal mass matrix $\mathcal{M}^2_{IJ}$. Since the adiabatic and the isocorvature modes are a linear combination of Q_ϕ and Q_χ, at horizon crossing a nonvanishing correlation between the adiabatic and the isocurvature modes is left imprinted in the spectrum.

4 Initial conditions in the radiation era

It must be emphasized that the mechanism described above for the production of adiabatic plus isocuvarture perturbations and their cross-correlation is active *during inflation*. A common feature of isocurvature perturbations is that they might not

survive after the end of inflation [28, 29, 30]. If during reheating all the scalar fields decay into the same species (photons, neutrinos, cold dark matter and baryons), the only remaining perturbations will be of adiabatic t

ype.

In fact, in order to have isocurvature perturbations deep in the radiation era it is necessary to have at least one non-zero isocurvature perturbation [31]:

$$S_{\alpha\beta} \equiv \frac{\delta_\alpha}{1+w_\alpha} - \frac{\delta_\beta}{1+w_\beta} \neq 0, \tag{53}$$

where $\delta_\alpha = \delta\rho_\alpha/\rho_\alpha$, $w_\alpha = p_\alpha/\rho_\alpha$ (the ratio of the pressure to the energy density), and α and β stand for any two components of the system (they can also be scalar fields). $S_{\alpha\beta}$ (a gauge-invariant quantity) measures the relative fluctuations between the different components. Adiabatic perturbations are characterized by having $S_{\alpha\beta} = 0$ for all of the components.

So in the following we will assume that the mixing between the scalar fields is negligible after inflation and that, for example, the scalar field ϕ decays into "ordinary" matter (photons, neutrinos and baryons) and the scalar field χ decays only into cold dark matter (or χ does not decay, like the axion, so that it consitutes the cold dark matter). In this case we can write:

$$\delta_{CDM} = S_{CDM-\mathrm{rest}} + \delta_A\,, \qquad \delta_A = \frac{3}{4}\delta_\gamma = \frac{3}{4}\delta_\nu = \delta_b \tag{54}$$

where δ_A specifies the amplitude of the adiabatic mode of perturbations, and "rest" stands fo photons, neutrinos and baryons.

In fact the initial conditions for the evolution of cosmological perturbations are set once $S_{\alpha\beta}$ and the gravitational potential Φ (in the longitudinal gauge) are given deep

in the radiation era [7]. These initial conditions are necessary to explore the effects on the large scale structure of the universe and the CMB anisotropies from correlated adiabatic/isocurvature perturbations [7, 8].

Here we want to show how to link the value of $S_{CDM-\mathrm{rest}}$ and Φ to the inflationary quantities δs and Q_A.

For the gravitational potential the main equation is:

$$\mathcal{R} = -\frac{H}{\dot{H}}\dot{\Phi} + \left(1 - \frac{H^2}{\dot{H}}\right)\Phi, \tag{55}$$

where $\mathcal{R}$ is the curvature perturbation (see [11], and reference therein).

Now, making an expansion in the slow-roll parameters to lowest order, it can be shown that the term proportional to $\dot{\Phi}$ can be neglected for most of the inflationary period. During the radiation dominated epoch $-H^2/\dot{H} = 1/2$, and by matchi

ng the two stages we can write:

$$\mathcal{R}_{\mathrm{rad}} = \frac{3}{2}\Phi, \tag{56}$$

where $\mathcal{R}_{\mathrm{rad}}$ is the curvature perturbation at the end of inflation, and it is directly related to Q_A through [11]:

$$\mathcal{R} = Q_A \frac{H}{\dot{A}} \quad , \qquad \dot{A} = (\cos\beta)\dot{\phi} + (\sin\beta)\dot{\chi}\,. \tag{57}$$

As far as $S_{CDM-\mathrm{rest}}$ is concerned, let us define the following quantity:

$$\delta_{\chi\phi} \equiv \frac{\delta\chi}{\dot{\chi}} - \frac{\delta\phi}{\dot{\phi}}\,. \tag{58}$$

For the two scalar fields ϕ and χ the isocurvature perturbation $S_{\chi\phi}$ as defined in Eq. (53) results to be $S_{\chi\phi} = a^3(\delta_{\chi\phi}/a^3)^{\cdot}$ [32].

On the other hand:

$$\delta s = \frac{\dot{\chi}\,\dot{\phi}}{\sqrt{\dot{\chi}^2 + \dot{\phi}^2}}\,\delta_{\chi\phi}\,. \tag{59}$$

Then, to lowest order in the slow-roll parameters and taking into account only the growing isocurvature mode, one finds:

$$S_{\chi\phi} = -\frac{3}{\sqrt{2}\,M}\,\frac{\sqrt{\varepsilon_{\phi\phi} + \varepsilon_{\chi\chi}}}{\varepsilon_{\phi\chi}}\,\delta s\,. \tag{60}$$

To match to the radiation epoch we take $S_{CDM-\mathrm{rest}} = S_{\chi\phi}$ at the end of inflation.

Finally, we can give an expression for the correlation between Φ and $S_{CDM-\mathrm{rest}}$ which represents the correlation between the adiabatic and isocurvature modes:

$$\langle \Phi(k) S^*(k') \rangle = -\frac{1}{8\pi\, M^2 \varepsilon_{\phi\chi}} \langle Q_A(k) \delta s^*(k') \rangle, \tag{61}$$

where the right-hand side of this equation is evaluated at the end of inflation, and $M = M_{\mathrm{Pl}}/\sqrt{8\pi}$.

In a separate paper [33] we calculate in detail the amplitudes and spectral indeces in terms of the slow-roll parameters for Φ, $S_{\alpha\beta}$ and the correlation.

5 Conclusions

In this paper we have shown that the correlation between adiabatic and entropy perturbations during inflation [7, 11] can be explained through an oscillation mechanism between the perturbations of two or more scalar fields.

If the isocurvature perturbation mode survives after inflation, this correlation can leave peculiar imprints on the CMB anisotropies as an additional parameter for the initial conditions of structure formation. In the near future very accurate data on CM B anisotropies will be available from the MAP [34] and Planck [35] experiments. Thus it is worth investigating further these issues, since they can represent a valid alternative to the simplest inflationary models. The latter are based on a single slow-rolling scalar field and generally predict adiabatic, scale-free and nearly Gaussian perturbations.

In this respect the relevant point of this work is to stress how isocurvature perturbations and cross correlations come out in a very natural way *even* within a single-field inflationary scenario, with an inflaton ϕ which gives the dominant con tribution to the total energy density and other scalar fields whose energy densities may or may not be subdominant. Moreover, the same correlation can give rise to a transfer of possible non-Gaussian features from the isocurvature mode to the adiabatic one. Different infla-

tionary models, in fact, have been proposed in the literature which predict non-Gaussian isocurvature perturbations [13]. In the present case, however, even if the isocurvature component is suppressed, as the observations suggest [9], this transfer mechanism can be very efficient for producing non-Gaussian *adiabatic* perturbations. We will investigate this possibility in a separate paper [36].

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